# Analytical higher-order theory for detailed stress and strain analyses of sandwich panels with flexible core

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## Introduction

Since the period after the Second World War a large variety of analytical solutions for sandwich panels loaded in bending has been proposed. Most of the classical theories, superposition approaches and higher-order theories are based on a three-layer concept. Each derivation makes assumptions in modelling the behaviour of the core, the faces and the interaction between both. This will result into a set of differential equations, which have to be solved analytically or with a numerical calculation procedure. To calculate the stress state near local geometrical details, the higher-order theory seems to be the most promising of the three distinguished categories of analytical solutions.

Compared with classical theories, higher-order theories take into account the fact that the normal stresses deviates over the core thickness and that there is a zero-stress condition at a free sandwich end. This means that the deflections of the upper and lower face are no longer equal and that the section planes are no longer linear for local circumstances. As a result a more realistic stress state can be determined for sandwich panels with a flexible core material.

Superposition approaches are not adequate alternatives for higher-order theories, because the local effects are only combined with classical theories without taking interaction effects into account. Another advantage of the higher-order theories is that the boundary conditions are described properly. Classical theories assume that the boundary conditions are the same for the entire height of the section, which is not very realistic for practical applications. To make use of the opportunities of higher-order theories, solutions for various applications have been developed by Frostig from the end of the 1980's on.

## Publications

Various publications of Frostig about higher-order theories from 1992 on are related to the considered case study of a sandwich panel under four-points bending. The derivations presented and discussed in [1] to [5] focus on sandwich beams. The faces are considered as ordinary beams, which are interconnected through equilibrium and compatibility at the interface layer with the core. The core is considered to be a two-dimensional elastic medium. Different boundary conditions and continuity requirements for the two faces and the core are allowed and different continuous and local loadings may be applied on the faces. With these derivations it is possible to analyse sandwich beams with, see also figure 1:
- point loads and support regions [1];
- edge and inner delamination regions [2];
- edge and inner transverse diaphragms [3];
- cut-off edge connections [4].

A summary of these derivations is given in [4], while in [5] an experimental verification is made for the case of a sandwich beam under three-point bending. A further development is made in [6] for a sandwich panel, which can bend in two directions.

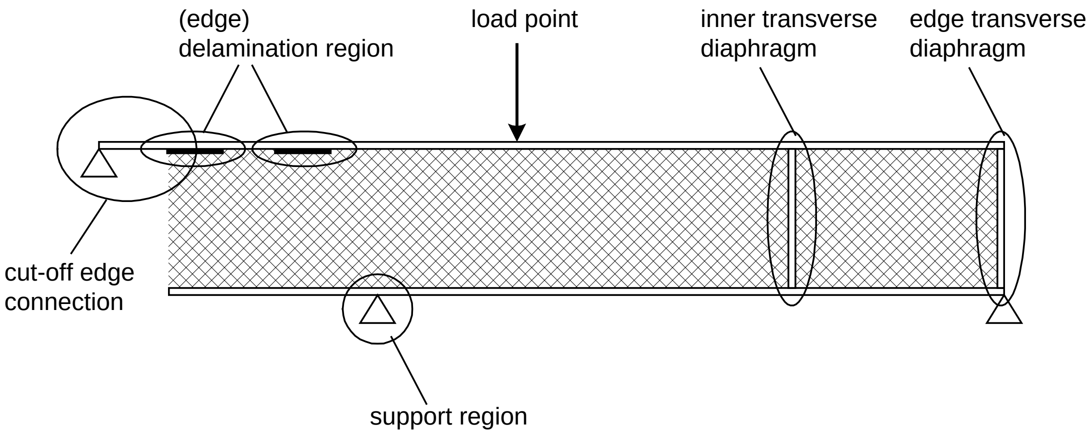

Figure 1 - Overview of a sandwich beams with local geometry for which higher-order theories are available [1] to [5]

## Backgrounds

The principle of a higher-order theory for modelling a beam, a plate, a shell or a sandwich is that the displacement fields as described by a classical theory, are extended by higher-order terms. The essential contribution of Frostig to the derivation of a higher-order theory for sandwich panels is that the derivations are based on the actual physical behaviour of the flexible core material, instead of simply adding higher-order terms.

The higher-order theory derived by Frostig can be seen as an extension of the classical theories. To model the mechanical behaviour of a sandwich panel a general procedure of derivation is followed. The kinematic relations of the faces and core, the compatibility conditions of the interface layer, the equilibrium equations of the interface layer and the constitutive relations of the faces and core have to be described. A summary of this procedure is given in figure 2.

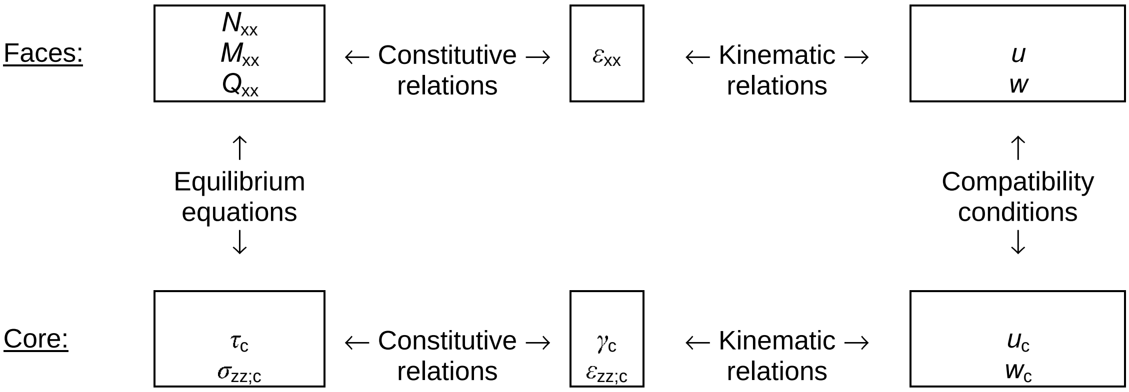

Figure 2 - General procedure of derivation

The main differences in derivation with the classical theories are the formulation of the kinematic relations of the core material and the possibility to meet the zero-shear stress condition at the free core ends. The kinematic relations of the core are formulated such that the transverse normal stress $\sigma_{zz;c}$ varies linear over the core height. A quadratic (second-order) transverse displacement field $w_c$ and a third-order longitudinal displacement field $u_c$ over the core height is derived. The following assumptions are made:

- Cylindrical bending of the faces. Both the beam theory as described by Frostig, as well as the plate theory might be used.
- Elastic isotropic material behaviour of faces and core.

- The effect of shear deformation of the faces is not taken into account, because it is assumed that the effects are negligible.
- Zero shear stress condition at the free core's ends.
- Constant shear stress over the core height.
- No longitudinal normal stress in the core.

The derivations of the differential equations, continuity and boundary conditions are originally presented by Frostig [1] to [5].

The results of these derivations are a set of second and fourth order differential equations and matching continuity and boundary conditions. The derived boundary value problem is of the 14th order. The set of differential equations contains the unknown functions of the displacements of the faces (longitudinal displacement in the centre line $u_o$, its first derivative, the transverse displacement $w$ and its first, second and third derivatives) and the shear stress in the core ($\tau_c$ and its first derivative). The matching continuity and boundary conditions are also formulated within these displacements and stresses. The section forces in the faces (normal force $N_{xx}$, shear force $Q_{xx}$ and bending moment $M_{xx}$), transverse normal stress within the core ($\sigma_{zz;c}$) and displacement fields of the core (transverse displacement $w_c$ and longitudinal displacement $u_c$) are formulated in terms of the unknown functions.

The boundary value problem according to the higher-order theory can be solved with a numerical algorithm. The derived set of second and fourth order differential equations has to be rewritten as a set of first order differential equations. Together with the boundary and continuity conditions this set can be solved with the multi-segment method of integration developed by Kalnins [7].

## Example of calculations

To illustrate the possibilities of the higher-order theory an example of a sandwich beam under 3-point bending is worked out. The set-up of the example is shown in figure 3. The following dimensions are used:

- beam height of 600 mm;
- beam width of 200 mm;
- sandwich height of 80 mm;
- thickness of upper face of 1.0 mm;
- thickness of lower face of 0.7 mm.

The following properties of the steel faces and heavy polystyrene core material are used:

- Young's modulus faces of 210000 N/mm$^2$;
- shear modulus core material of 20 N/mm$^2$;
- Young's modulus core material of 60 N/mm$^2$.

The applied load is equal to 3000 N.

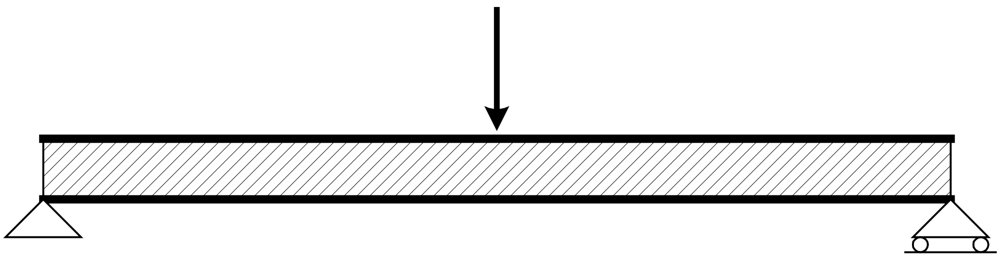

Figure 3 - Example of sandwich beam under 3-point bending

To simplify the calculations, only the right half of the geometry is considered. The calculated deflections of both the upper and the lower faces of the sandwich are presented in figure 4. These results show the significant influence of the local effects of the applied force and the support. To show the detailed information provided by the higher-order theory, the calculated vertical and horizontal deformations of the core material are plotted in figure 5. The significant difference between the load point region and the support region is cased by the fact that near the load point the upper faces is continuous, while for the support region the lower face ends.

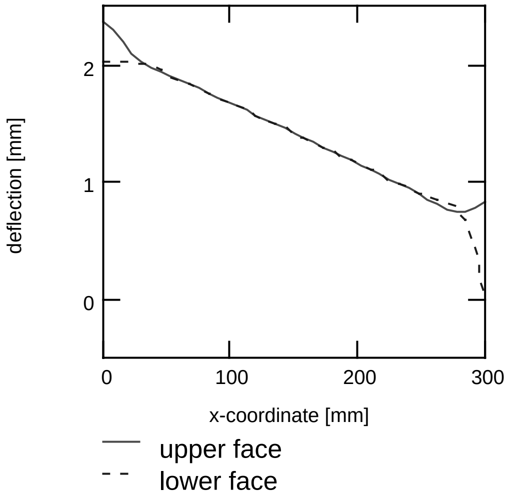

Figure 4 - Deflections of upper and lower faces; the x-coordinate equal to 0 is near the load point, while the x-coordinate equal to 300 mm is near the support.

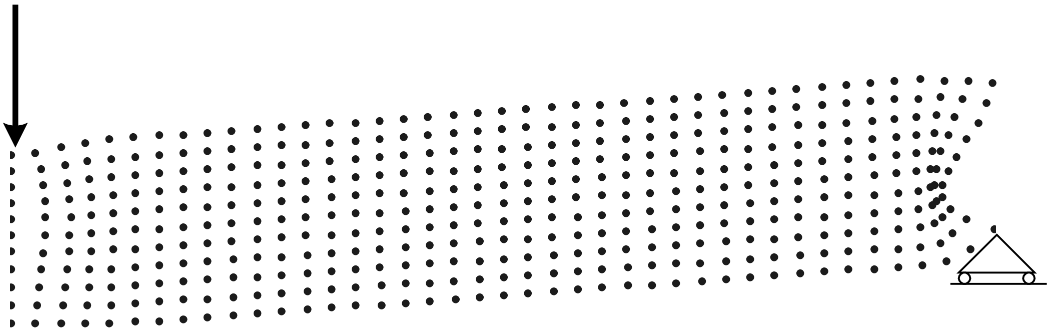

Figure 5 - Magnified deformations of the core material.

The stresses in the upper and lower faces due to overall bending of the sandwich beam and local bending of the faces are presented in figure 6. It is observed that near the load point and support region the local bending stresses are high compared with the membrane stresses due to overall bending. The example illustrates that an advantage of the higher-order theory is that it is possible to investigate the effects in the unloaded faces due to a force active on the opposite face.

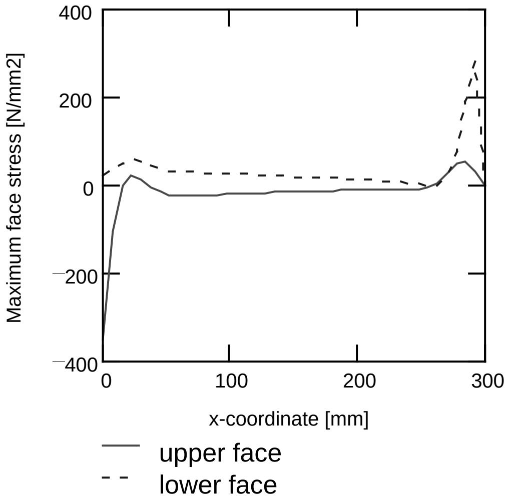

Figure 6 - Maximum stresses in faces

The distributions of the shear stresses and the peel (transverse normal) stresses in the interface between the core material and the faces are presented in figure 7. The higher-order theory is capable to show the influence of local effects on the shear stress distribution. The high peel stresses between the core material and the upper face in the support region might cause delamination of the interface.

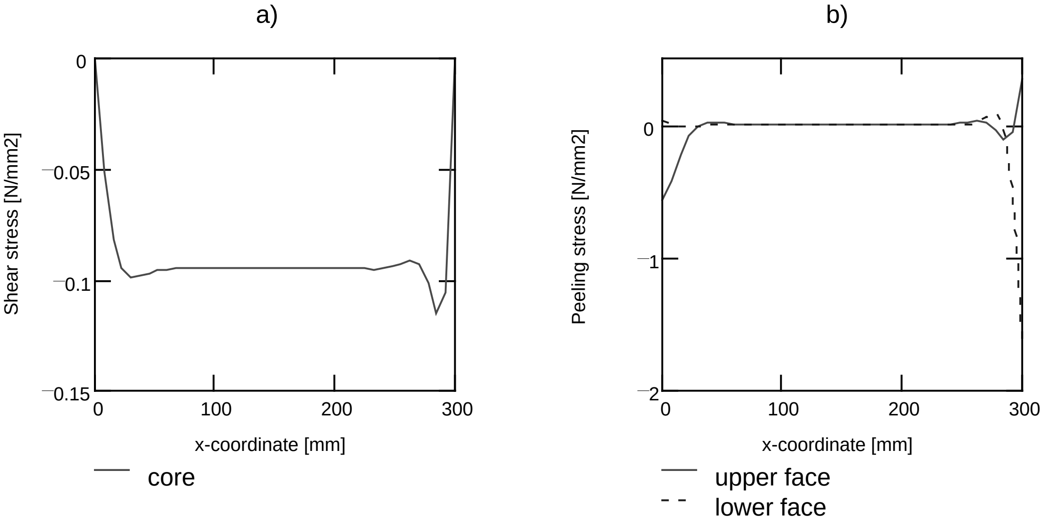

Figure 7 - Stress distributions: a) shear stresses in the core and b) peel stresses in the interface.

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