

LONG-TERM BEHAVIOUR PREDICTIONS POLYMERIC MATRIX COMPOSITE MATERIALS

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Abstract

Polymer based composite systems are nowadays widespread in many industrial applications, from aerospace, aeronautical, naval, automobile, building to medical prosthesis. Nevertheless these materials are limited in these applications due to the lack of confidence in the prediction of the residual properties in a long-term basis, i.e., for a given loading history and environmental conditions. Much work have been done to overcome these difficulties, improving the theoretical models and establishing efficient and reliable experimental methodologies to determine the relevant properties.

Polymers exhibit a time-dependent behavior as a consequence of its viscoelastic nature. The composite systems due to the advanced fiber reinforcement form a high-performance structural material. Reinforcements, as carbon fibers, exhibit a negligible viscoelastic behavior and even when combined with a viscoelastic matrix that results in plies which exhibit very small amount of creep or stress relaxation in the reinforcement direction. However these plies may exhibit considerable amounts of creep in transverse and shear directions. When these plies are combined to form a laminate it results in a structure that exhibit creep, stress relaxation and delayed failure.

Keywords

Creep, stress relaxation, viscoelasticity, viscoplasticity.

1. A MODEL FOR THE LONG TERM PREDICTIONS

In order to predict the long term behaviour of the basic material component, a research program based on the idea proposed by Hal Brinson (Brinson, 1978) was developed. The basis of the method of accelerating factors in viscoelastic and viscoplastic models was the work done by Schapery (Schapery, 1966). The accelerating factors considered were the stress and the temperature. In future more factors should be included such as the moisture, the ageing effects and the mechanical degradation. The prediction of the long-term durability of composites must be based on the coupled effects of all these nonlinear mechanisms. A brief discussion of that research program is given bellow.

The single integral formulations, such as the Schapery nonlinear viscoelastic theory, thought simple in form, have been found to be mathematically too complex to be used in an engineering analysis, due to the presence of Volterra-type integrals. Tuttle and Brinson (1986) presented a closed-form solution to an arbitrary number of discrete steps in stress based upon the Schapery theory. This solution, based on hereditary nature, requires that all stresses at each time step are stored and reused for all subsequent time steps. Furthermore, each additional time step requires the hereditary integral to be recalculated from the initial starting time. Therefore, the total solution time grows geometrically with the number of time steps that ultimately limits the number of time steps, regardless the size of the step or the speed of the computer. An efficient method to handle the constitutive integral equation of Schapery

theory is presented. The method avoids storing the stress history and computes the Volterra-type integral by replacing it with numerical equivalent ordinary integrals. The proposed method, to handle the constitutive integral equations of Schapery theory, can be easily used to predict the stress relaxation and the rate-dependent stress/strain behaviour.

2. ANALYTICAL-MODEL DESCRIPTION

The predictions for multiple-angle laminates can be obtained using the classical lamination theory. The constitutive models are defined at the ply level, which is a "macromechanic" approach. Let the plane of a single ply be defined by the 1-2 coordinated system, where the 1 and 2 axes are parallel and perpendicular to the fibres, respectively. Consider a ply subjected to a plane stress state $\{\sigma_{11}, \sigma_{22}, \tau_{12}\}$ that may change with the time. The total strains induced within the ply are given by

$$\begin{Bmatrix} \varepsilon_{11}(t) \\ \varepsilon_{22}(t) \\ \gamma_{12}(t) \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22}(t) & 0 \\ 0 & 0 & S_{66}(t) \end{bmatrix} \begin{Bmatrix} \sigma_{11}(t) \\ \sigma_{22}(t) \\ \tau_{12}(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ \varepsilon_{22}(t) \\ \gamma_{12}(t) \end{Bmatrix}_{VP} \quad (1)$$

where the subscript VP denotes the viscoplastic component of the strain. We assumed that the fibre-dominated compliance terms S_{11} and $S_{12}(=S_{21})$ are time-independent, while the matrix dominated compliance terms $S_{22}(t)$ and $S_{66}(t)$ are time-dependent.

The total strains associated with the matrix-dominated compliances S_{22} and S_{66} were modelled using the modified Schapery theory to include the viscoplastic behaviour, firstly presented by Tuttle (1993). According to this, the total ply strains induced by an arbitrary stress history are given by

$$\begin{Bmatrix} \varepsilon_{11}(t) \\ \varepsilon_{22}(t) \\ \gamma_{12}(t) \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & g_{0,22}S_{0,22} & 0 \\ 0 & 0 & g_{0,66}S_{0,66} \end{bmatrix} \begin{Bmatrix} \sigma_{11}(t) \\ \sigma_{22}(t) \\ \tau_{12}(t) \end{Bmatrix} + \begin{Bmatrix} 0 \\ g_{1,22} \int_0^t \Delta S_{22}(\psi - \psi') \frac{d(g_{2,22}\sigma_{22}(\tau))}{d\tau} d\tau \\ g_{1,66} \int_0^t \Delta S_{66}(\psi - \psi') \frac{d(g_{2,66}\tau_{12}(\tau))}{d\tau} d\tau \end{Bmatrix} + \begin{Bmatrix} 0 \\ \left\{ C_{22} \int_0^t \sigma_{22}^{N_{22}}(\tau) d\tau \right\}^{n_{22}} \\ \left\{ C_{66} \int_0^t \tau_{12}^{N_{66}}(\tau) d\tau \right\}^{n_{66}} \end{Bmatrix} \quad (2)$$

where C_{22}, N_{22}, n_{22} and C_{66}, N_{66}, n_{66} are stress-independent but temperature dependent material properties. The kernels $\Delta S_{22}(t)$ and $\Delta S_{66}(t)$ are the transverse and shear elastic compliance, respectively, with the correspondent reduced times Ψ and Ψ' given by

$$\Psi = \int_0^t \frac{d\tau'}{a_{\sigma,22}} \quad , \quad \Psi' = \int_0^{\tau} \frac{d\tau'}{a_{\sigma,22}} \quad (3)$$

and

$$\Psi = \int_0^t \frac{d\tau'}{a_{\sigma,66}} \quad , \quad \Psi' = \int_0^{\tau} \frac{d\tau'}{a_{\sigma,66}} \quad (4)$$

where $g_{0,22}, g_{1,22}, g_{2,22}, a_{\sigma,22}$ and $g_{0,66}, g_{1,66}, g_{2,66}, a_{\sigma,66}$ are stress-dependent nonlinearizing parameters. The common parameter for the transverse and shear nonlinear compliance is the octahedral shear stress in the matrix that is a function of matrix transverse stress and matrix shear stress. A more detailed explanation of the octahedral shear stress parameter is given by Schapery (1969).

In order to eliminate the Volterra-type integrals, the transverse and shear compliance are expressed using Prony series, as Gramoll (1989) and Czyz (1990) have already discussed.

$$\begin{cases} \Delta S_{22}(t) = \sum_{i=1}^{\infty} S_{i,22} (1 - e^{-\lambda_{i,22}t}) \\ \Delta S_{66}(t) = \sum_{i=1}^{\infty} S_{i,66} (1 - e^{-\lambda_{i,66}t}) \end{cases} \quad (5)$$

where $S_{i,22}, \lambda_{i,22}, S_{i,66}, \lambda_{i,66}$ are linear viscoelastic parameters for the transverse and shear compliance, respectively. For example substituting **Error! Reference source not found.** into **Error! Reference source not found.** for the transverse strain, we obtain

$$\varepsilon_{22}(t) = g_{0,22} S_{0,22} \sigma_{22}(t) + g_{1,22} \sum_{i=1}^{\infty} \varepsilon_{i,22}(t) + \varepsilon_{vp,22}(t) \quad (6)$$

where

$$\varepsilon_{i,22}(t) = S_{i,22} \int_0^t (1 - e^{-\lambda_{i,22}(\psi - \psi')}) \frac{dg_{2,22} \sigma_{22}}{d\tau} d\tau \quad (7)$$

After some transformations, equation **Error! Reference source not found.**, furnishes the relation between the internal strain $\varepsilon_{i,22}(t_{j+1})$ and $\varepsilon_{i,22}(t_j)$. Let assume a linear variation during $t_j \leq t \leq t_{j+1}$ for the stress, σ_{22} , as

$$\frac{d\sigma_{22}}{dt} = \frac{\sigma_{22}(t_{j+1}) - \sigma_{22}(t_j)}{\Delta t} \quad (8)$$

Let also assume that the function $\tilde{g}_{i,22}(\sigma)$ is also approximated by a linear variation as

$$\frac{d\tilde{g}_{i,22}(\sigma)}{dt} \cong \frac{\tilde{g}_{i,22}(\sigma_{22}(t_{j+1})) - \tilde{g}_{i,22}(\sigma_{22}(t_j))}{\Delta t} \quad (9)$$

where

$$\tilde{g}_{i,22}(\sigma) = S_{i,22} g_{2,22} \sigma_{22}$$

(10)

Substituting **Error! Reference source not found.** into **Error! Reference source not found.** we obtain

$$\varepsilon_{i,22}(t_j) = \tilde{g}_{i,22}(\sigma_{22}(t_j)) - e^{-\lambda_{i,22}\theta(t_j)} \int_0^{t_j} e^{-\lambda_{i,22}\theta(\tau)} \frac{d\tilde{g}_{i,22}(\sigma)}{d\tau} d\tau \quad (11)$$

where $\theta(t) = \int_0^t \frac{d\tau'}{a_{\sigma,22}}$.

In order to calculate $\varepsilon_{i,22}(t_{j+1})$ in terms of $\varepsilon_{i,22}(t_j)$. we obtain

$$\begin{aligned} \varepsilon_{i,22}(t_{j+1}) = & \tilde{g}_{i,22}(\sigma_{22}(t_{j+1})) - e^{-\lambda_{i,22}[\theta(t_{j+1})-\theta(t_j)]} [\tilde{g}_{i,22}(\sigma_{22}(t_j)) - \varepsilon_{i,22}(t_j)] \\ & - \int_{t_j}^{t_{j+1}} e^{-\lambda_{i,22}[\theta(t_{j+1})-\theta(\tau)]} \frac{d\tilde{g}_{i,22}(\sigma)}{d\tau} d\tau \end{aligned} \quad (12)$$

The integral of Equation **Error! Reference source not found.** can be solved using the approximation function **Error! Reference source not found.** as

$$\begin{aligned} & \int_{t_j}^{t_{j+1}} e^{-\lambda_{i,22}[\theta(t_{j+1})-\theta(\tau)]} \frac{d\tilde{g}_{i,22}(\sigma)}{d\tau} d\tau = \\ & = \frac{\tilde{g}_{i,22}(\sigma_{22}(t_{j+1})) - \tilde{g}_{i,22}(\sigma_{22}(t_j))}{\Delta t} \cdot \frac{a_{\sigma,22}(\sigma_{22}(t_{j+1}))}{\lambda_{i,22}} \left(1 - e^{-\lambda_{i,22} \frac{\Delta t}{a_{\sigma,22}(\sigma_{22}(t_{j+1}))}} \right) \end{aligned} \quad (13)$$

where $\Delta t = t_{j+1} - t_j$ and the following approximation was used

$$-\lambda_{i,22}[\theta(t_{j+1})-\theta(\tau)] = -\lambda_{i,22} \int_{\tau}^{t_{j+1}} \frac{d\tau'}{a_{\sigma,22}(\sigma_{22})} \cong \frac{\lambda_{i,22}}{a_{\sigma,22}(\sigma_{22}(t_{j+1}))} (t_{j+1} - \tau) \quad (14)$$

Substituting (10) into (9) we obtain

$$\begin{aligned} \varepsilon_{i,22}(t_{j+1}) = & e^{-\eta_i \Delta t} \cdot \varepsilon_{i,22}(t_j) + \left[1 - \frac{1}{\eta_i \Delta t} (1 - e^{-\eta_i \Delta t}) \right] \tilde{g}_i(\sigma_{22}(t_{j+1})) \\ & + \left[\frac{1}{\eta_i \Delta t} (1 - e^{-\eta_i \Delta t}) - e^{-\eta_i \Delta t} \right] \tilde{g}_i(\sigma_{22}(t_j)) \end{aligned} \quad (15)$$

where

$$\eta_i = \frac{\lambda_{i,22}}{a_{\sigma,22}(\sigma_{22}(t_{j+1}))} \quad (16)$$

Tuttle (1993) presented the solution of the viscoplastic term assuming a linear variation in stress over every time step as

$$\begin{aligned}\varepsilon_{22}(t_{j+1})_{VP} &= \left\{ C_{22} \int_0^{t_j} [\sigma_{22}(\psi)]^{N_{22}} d\psi + C_{22} \int_{t_j}^{t_{j+1}} [\sigma_{22}(\psi)]^{N_{22}} d\psi \right\}^{n_{22}} \\ &= \left\{ \beta(t_j) + C_{22} \frac{[\sigma_{22}(t_{j+1})]^{N_{22}+1} - [\sigma_{22}(t_j)]^{N_{22}+1}}{(N_{22}+1)[\sigma_{22}(t_{j+1}) - \sigma_{22}(t_j)]} (t_{j+1} - t_j) \right\}^{n_{22}}\end{aligned}\quad (17)$$

with

$$\beta(t_j) = \beta(t_{j-1}) + C_{22} \frac{\sigma_{22}(t_{j+1})^{N_{22}+1} - \sigma_{22}(t_j)^{N_{22}+1}}{(N_{22}+1)[\sigma_{22}(t_{j+1}) - \sigma_{22}(t_j)]} (t_j - t_{j-1}) \quad (18)$$

The total strain for each layer, given by Eq. (2), can then be written as

$$\{\varepsilon(t_{j+1})\} = [S_{elast}] \{\sigma(t_{j+1})\} + \{R(t_{j+1})\} \quad (19)$$

where

$$\{R(t_{j+1})\} = \begin{Bmatrix} 0 \\ g_{1,22} \sum_{i=1}^n \varepsilon_{i,2}(t_{j+1}) + \varepsilon_{vp,2}(t_{j+1}) \\ g_{1,22} \sum_{i=1}^n \varepsilon_{i,6}(t_{j+1}) + \varepsilon_{vp,6}(t_{j+1}) \end{Bmatrix} \quad (20)$$

The vector of Equation **Error! Reference source not found.** contains the viscoelastic and viscoplastic deformations as given by Equation **Error! Reference source not found.**. This vector is calculated recursively using the formulas given by Eqs. **Error! Reference source not found.** and **Error! Reference source not found.** since they can equally be applied into transverse and shear deformations.

The objective is to determine the behaviour of multidirectional composites knowing the stress-strain relationships for unidirectional plies. Therefore a modified laminate plate theory for laminates subjected to in-plane loads $\{N\}$ and out-plane-loads $\{M\}$ was developed. The subscript (x, y, z) represents the off-axis coordinate directions (laminate or global coordinates).

Assuming that the ply strain is linear in the thickness coordinate z , the total strain of the ply k in the global coordinate system is given by

$$\{\varepsilon(t_{j+1})\}_k = \begin{Bmatrix} \varepsilon_x^0(t_{j+1}) \\ \varepsilon_y^0(t_{j+1}) \\ \gamma_{xy}^0(t_{j+1}) \end{Bmatrix} + z_k \begin{Bmatrix} k_x(t_{j+1}) \\ k_y(t_{j+1}) \\ k_{xy}(t_{j+1}) \end{Bmatrix}$$

(21)

where $\{\epsilon^0\}$ is the in-plane laminate strain vector, $\{k\}$ is the laminate curvature vector and z_k represents the z coordinate of the k ply. Taking into account the in-plane loads and moments acting on the laminate, the equilibrium equations in global coordinates are

$$\begin{cases} \int_{-h/2}^{h/2} \{\sigma\} dz = \{N\} \\ \int_{-h/2}^{h/2} \{\sigma\} z dz = \{M\} \end{cases} \quad (22)$$

where h represents the total laminate thickness. If we assume a constant stress at each ply, which is not true for flexural loads, using Eqs. **Error! Reference source not found.**, **Error! Reference source not found.** and **Error! Reference source not found.** we obtain a system of equations that allow to determine the in-plane laminate strain, the laminate curvature and the stress state in each ply as

$$\begin{cases} \{\epsilon^0\} + \{k\}z_1 - [S_{elast}]_1 \{\sigma\}_1 = \{R\}_1 \\ \{\epsilon^0\} + \{k\}z_2 - [S_{elast}]_2 \{\sigma\}_2 = \{R\}_2 \\ \vdots \\ \{\epsilon^0\} + \{k\}z_p - [S_{elast}]_p \{\sigma\}_p = \{R\}_p \\ \{\sigma\}_1 z_1 + \{\sigma\}_2 z_2 + \dots + \{\sigma\}_p z_p = \{N\} \\ \{\sigma\}_1 h_1 z_1 + \{\sigma\}_2 h_2 z_2 + \dots + \{\sigma\}_p h_p z_p = \{M\} \end{cases} \quad (23)$$

where the subscripts indicate the ply number. Since the vector $\{R\}_k$ depends on the present stress state of the k ply, an iterative procedure for each time step must be used until the stress state converges.

In order to avoid large systems of equations the ply stress state given by Eq. **Error! Reference source not found.** can be used in Eq. **Error! Reference source not found.** to obtain the following condensed system of equations as

$$\begin{bmatrix} \sum_{k=1}^p [S_{elast}]_k^{-1} z_k & \sum_{k=1}^p [S_{elast}]_k^{-1} z_k h_k \\ \sum_{k=1}^p [S_{elast}]_k^{-1} z_k h_k & \sum_{k=1}^p [S_{elast}]_k^{-1} \left(z_k^2 h_k + \frac{h_k^3}{12} \right) \end{bmatrix} \begin{Bmatrix} \{\epsilon^0\} \\ \{k\} \end{Bmatrix} = \begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} + \begin{Bmatrix} \sum_{k=1}^p [S_{elast}]_k^{-1} \{R\}_k z_k \\ \sum_{k=1}^p [S_{elast}]_k^{-1} \{R\}_k z_k h_k \end{Bmatrix} \quad (24)$$

where h_k represents the thickness of k ply. The previous formulation permit us to solve the creep problem. If a restriction is imposed, like prescribing an in-plane strain or a curvature, then the relaxation problem must be solved. In a creep test the loads are imposed and the in-plane strain and curvature are determined. In a relaxation test the in-plane strain and/or

curvature are imposed and the resulting loads are to be determined. The present method can also be used to solve these type of problems. For example, if the k_x curvature is prescribed, then the related flexural load M_x is to be determined. The problem is solved by exchanging M_x with k_x in Eq. **Error! Reference source not found.**, resulting in the following system of equations

$$\left\{ \begin{array}{l} \left\{ \varepsilon^0 \right\} + \begin{Bmatrix} 0 \\ k_y \\ k_{xy} \end{Bmatrix} z_1 - [S_{elast}]_1 \left\{ \sigma \right\}_1 = \left\{ R \right\}_1 - \begin{Bmatrix} k_x \\ 0 \\ 0 \end{Bmatrix} z_1 \\ \vdots \\ \left\{ \varepsilon^0 \right\} + \begin{Bmatrix} 0 \\ k_y \\ k_{xy} \end{Bmatrix} z_p - [S_{elast}]_p \left\{ \sigma \right\}_p = \left\{ R \right\}_p - \begin{Bmatrix} k_x \\ 0 \\ 0 \end{Bmatrix} z_p \\ \\ \left\{ \sigma \right\}_1 z_1 + \left\{ \sigma \right\}_2 z_2 + \dots + \left\{ \sigma \right\}_p z_p = \left\{ N \right\} \\ \left\{ \begin{array}{l} M_x \\ 0 \\ 0 \end{array} \right\} + \left\{ \sigma \right\}_1 h_1 z_1 + \left\{ \sigma \right\}_2 h_2 z_2 + \dots + \left\{ \sigma \right\}_p h_p z_p = \left\{ \begin{array}{l} 0 \\ M_y \\ M_{xy} \end{array} \right\} \end{array} \right. \quad (25)$$

The argument for the existing solution was already discussed before. In strict mathematical terms, each restriction implies an inversion of the correspondent nonlinear integral equation and numerical methods are the only way to obtain the solution. We can conclude that this formulation allows us to solve all sort of problems related with creep, relaxation and rate dependent stress/strain behaviour for in-plane and flexural loads. The method described here was developed in a FORTRAN computer program called LAMFLU.

3.MATERIAL PROPERTIES CALCULATION

A numerical procedure to fit the Schapery parameters was developed. Schapery and Lou (1971) described a mixed procedure, graphical and numerical, to determine all the parameters of Schapery model. In this case a full numerical approach was developed in order to avoid the inherent subjectivity of the graphical analysis. A plastic modified Schapery model was chosen as it was proposed and applied by Tuttle and Pasricha (Tuttle, 1995).

The material properties were obtained through a series of creep/creep-recovery tests. The stress history in a creep/creep-recovery consists of a constant stress (σ_0) applied during the creep phase followed by removing the total applied stress during the creep-recovery phase of the test. During creep time ($0 \leq t \leq t_a$) the strain response is given by

$$\begin{aligned} \varepsilon_{creep}(t) = & D_0 g_0 \sigma_0 + \\ & g_1 g_2 \sum_{i=1}^m D_i \left(1 - e^{-\frac{t}{\tau_i a_\sigma}} \right) \sigma_0 + \\ & \left\{ C \sigma_0^N t \right\}^n \end{aligned} \quad (26)$$

During the recovery time ($t > t_a$) the response is given by

$$\begin{aligned} \varepsilon_{recovery}(t) = & g_2 \sum_{i=1}^m D_i \left[e^{-\frac{t-t_a}{\tau_i}} \left(1 - e^{-\frac{t_a}{\tau_i a_\sigma}} \right) \right] \sigma_0 + \\ & \left\{ C \sigma_0^N t_a \right\}^n \end{aligned} \quad (27)$$

Data collected from creep/creep-recovery tests at several stress levels in combination with a fully numerical procedure were used to obtain the material properties.

In this method the problem was "linearized" by prescribing the $\{\tau_i\}_{i=1,2,\dots,M}$ parameters. The first step is the estimation of the irrecoverable viscoplastic strain induced during creep at various stress levels. The recovery curves given by Equation **Error! Reference source not found.** can be rewritten in the following compact form

$$\varepsilon_{recovery}(t) = A_0 + \sum_{i=1}^m A_i e^{-\frac{t-t_a}{\tau_i}} \quad (28)$$

where A_0, A_i are parameters to be determined for each stress level. Equation **Error! Reference source not found.** is numerically curve fit to the recovery data using one time constant per decade of time. From the previous formulation it is possible to extrapolate the recovery curves for very long times, calculating the limit

$$\lim_{t \rightarrow \infty} \varepsilon_{recovery}(t) = A_0 = \left\{ C \sigma_0^N t_a \right\}^n \quad (29)$$

The product $N \cdot n$ is determined by curve fitting of Equation **Error! Reference source not found.** From the lower stress level, assumed to be a linear level, the values of the linear viscoelastic compliance are determined using Equations **Error! Reference source not found.** and **Error! Reference source not found.** as

$$D_i = \frac{A_i}{\sigma_0 \left(1 - e^{-\frac{t_a}{\tau_i}} \right)} \quad (30)$$

The nonlinear stress levels are then used to determine the parameters g_2 and a_σ as

$$A_i - g_2 \sigma_0 D_i \left(1 - e^{-\frac{t_a}{a_\sigma \tau_i}} \right) = 0 \quad (31)$$

The values for D_0 , C and n are evaluated by fitting Equation **Error! Reference source not found.** to the linear recovery data. Finally the parameters g_0 and g_1 are evaluated by fitting Equation **Error! Reference source not found.** to the nonlinear creep data.

The linear viscoelastic creep compliance and the viscoplastic flow are the only time-dependent material properties in the present model. The other material properties depend on the stress and on the temperature. The material properties were expressed as functions of the octahedral shear stress in the matrix. The method described was developed into a FORTRAN computer program called XAPFIT.

4. EXPERIMENTAL RESULTS

Some experimental results are presented for tests conducted on composite materials under creep/creep-recovery, relaxation and ramp loading.

In Figure 2 shear strain history for the test data and the LAMFLU prediction are plotted for a T300/5208 [$\pm 45^\circ$] laminate. The cyclic creep/creep-recovery tests at room temperature were conducted for a total time of 10176 hours, representing approximately 14 months. A very good agreement between the prediction and measured data over the entire test was obtained for the theoretical model.

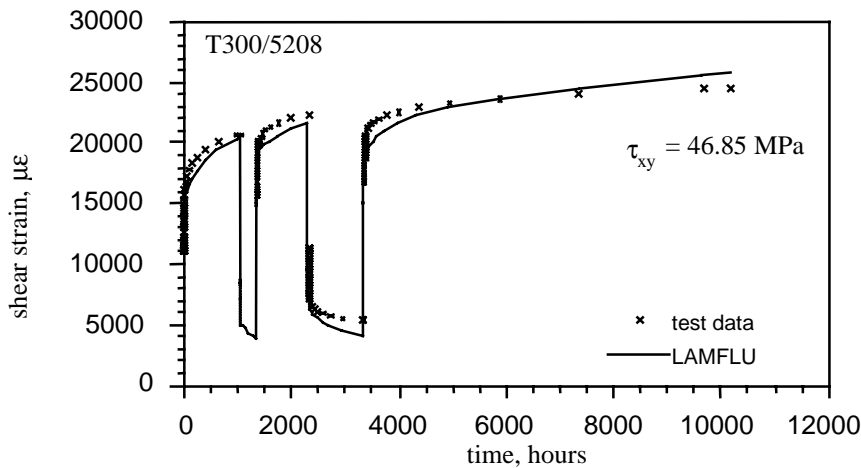


Fig. 2. Test data and LAMFLU prediction shear strain of a T300/5208 [$\pm 45^\circ$] laminate subjected to a cyclic creep/creep-recovery test at room temperature.

The LAMFLU procedure was used to compute the relaxation shear stress for different constant shear strains extending them into nonlinear range. The resulting relaxation behaviour predicted from the creep data showed a good agreement with the actual shear stress relaxation behaviour as shown in Fig. 3.

In Figure 4, test data and respective plastic shear strain are plotted for a shear strain rate of $1250\mu\text{s/s}$ together with the LAMFLU prediction. Comparison between the experimental result and the curve based on the theoretical model show an excellent agreement.

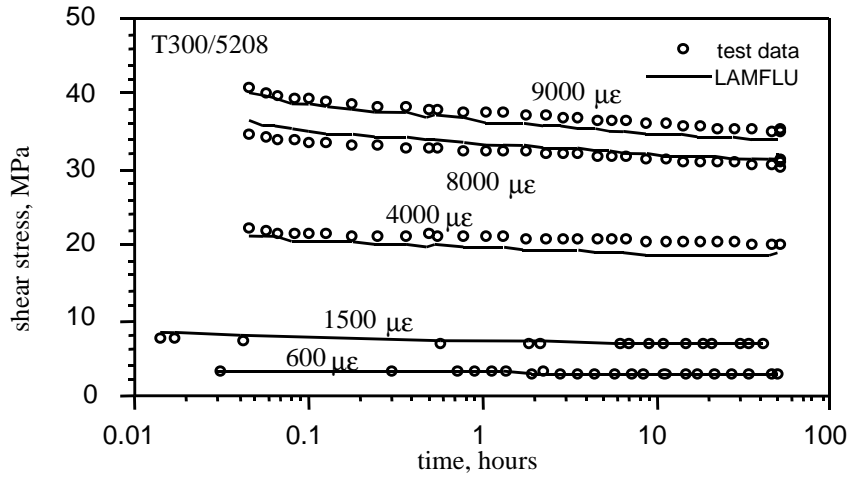


Fig. 3. Shear stress relaxation of T300/5208 [$\pm 45^\circ$] laminates under constant shear strain at room temperature, and predicted with LAMFLU.

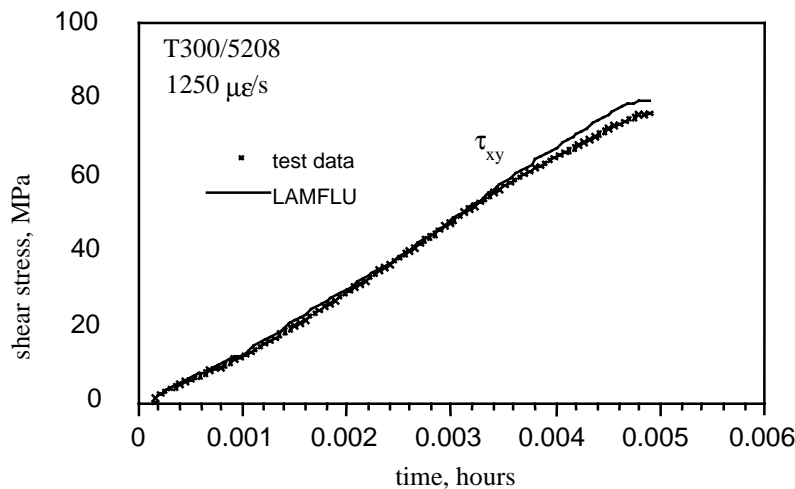


Fig. 4. Constant shear strain rate test of T300/5208 [$\pm 45^\circ$] laminate at room temperature, with those predicted with LAMFLU.

In Figure 5 the maximum strain history is plotted for four specimens in a four point bending test together with theoretical prediction. The test program consists on 6 cycles, 3 steps to increase the load followed by another 3 steps to decrease the load. Good agreement with experimental data was obtained for the first three load-steps. For the load decreasing phase, the LAMFLU predictions showed a fair agreement with experimental data with deviations lower than 15%.

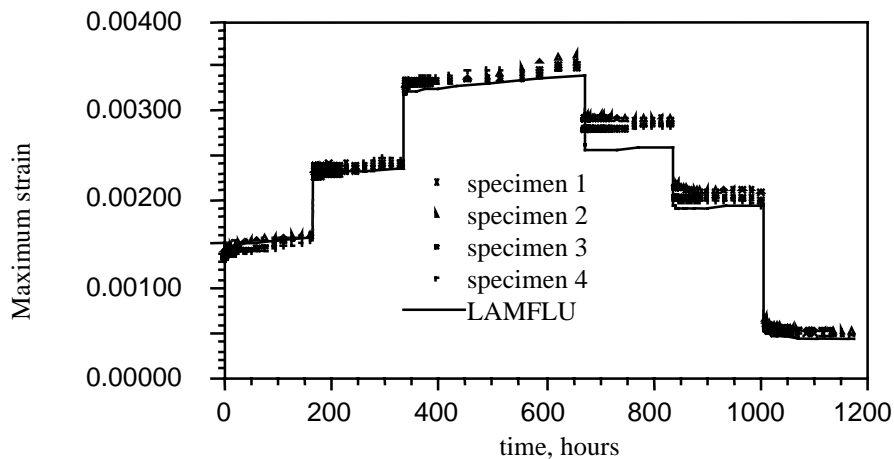


Fig. 5. Maximum strain results and theoretical predictions for four point bending tests.

CONCLUSION

A numerical method to handle the constitutive integral equation of the Schapery theory was developed. The procedure can be used to predict the creep for inplane loading and bending, the stress relaxation, the rate-dependent stress/strain behaviour of composite laminates. A good agreement between computed and experimental was observed in the presented examples. The plastic modified Schapery model proposed by Tuttle (1993) appear to be a general model at least for the limited stress or strain states and histories used in this study.

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