COMPREHENSIVE OVERVIEW OF THEORIES FOR SANDWICH PANELS

IJSBRAND J. VAN STRAALEN

TNO Building and Construction Research, P.O. Box 49, 2600 AA DELFT, The Netherlands

mail: IJ.vanStraalen@bouw.tno.nl

Abstract

This paper gives a comprehensive overview of theories for sandwich panels, dealing with the mechanical behaviour. The classical theories are mainly formulated during the fifties and sixties. The derived analytical solutions are based on simplified assumptions. To study local effects near supports, load points and other discontinuities, superposition approaches and higher-order theories have been developed during recent years. Mostly these derivations can only be solved with use of numerical solving techniques. Finite-element methods developed during the last two decades, are an alternative for the analytical solutions. The three-layer models seems to be very popular, but also more detailed three-dimensional models are usefull. One application of sandwich panels is discussed more in detail; for the building sector the development of design rules is reviewed. It is observed that the rules used nowadays, are mainly based on classical theories. To illustrate the advantages of the other theories in case of local sandwich behaviour, an example of a sandwich beam under three point bending is presented. This paper concludes with suggestions how to make use of these recently developed sandwich theories.

Keywords

Sandwich panels, classical theory, superposition approach, higher order theory, finite element method

1. INTRODUCTION

A main conclusion of a literature survey about sandwich panels is that the existing literature is either highly academic or scattered in short overviews in various textbooks and journals. For this reason researchers in Nordic countries have summarised existing knowledge about the design of sandwich panels in a suitable format recently [1]. Overviews of typical material properties, existing theories, design methods and research results are included in this recent state-of-the-art. The aim of this paper on the other hand is to focus on and to give a comprehensive overview of theories for sandwich panels, dealing with the mechanical behaviour.

Before discussing various theories, the types of sandwich panels and possible failure modes distinguished in literature are summarized. The proposed theories are classified and generally described. A detailed description of the assumptions, derivations and proposed solutions for each theory is out of the scope of this paper; for more information the reader is referred to the extended list of references. Specially the newly developed higher-order theories are of interest, because these are capable to deal with local behaviour and to give results in a short time.

It is noted here that it is a difficult task to draw up an overview of available theories. This is caused by the large number of developed theories, the complexity of the derivations, the differences between the approaches used by the experts and the fields of application. Another reason is that most recent state-of-the-arts of available theories are written during the sixties.

The available expertise is summarised in [2], [3] and [4], but since a number of new analytical approaches have been developed during the last two decades and finite element methods are more widely used, these general overviews are no longer representative. A more recent overview [5] refers to an almost unlimited number of publications (1376 in total!), but since the scope is mainly focused on finite element methods, it does not give a complete overview of existing analytical solutions.

2. TYPES OF SANDWICH PANELS AND POSSIBLE FAILURE MODES

2.1 Overall geometry

A sandwich panel is a composition of a "weak" core material with “strong and stiff” faces bonded on the upper and lower side as illustrated in figure 1. Different core materials are applied, like:

- honeycomb material;
- corrugated material;
- wood;
- expanded plastics (foam);
- mineral wool.

Also the faces can be made of different materials, like:

- thin metal plates;
- profiled plates;
- thick fibre reinforced composite materials.

The behaviour of the faces can be isotropic, orthotropic or anisotropic.

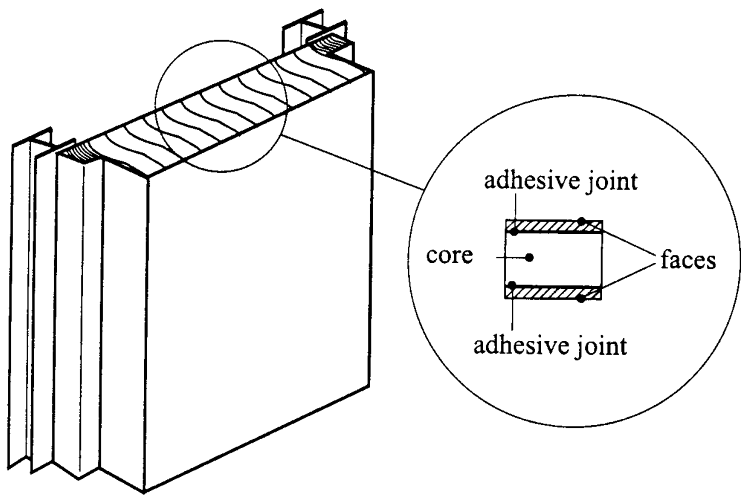


Figure 1 - Sandwich panel.

In general the following distinction is made between different types of sandwich panels:

- Sandwich beams. A large number of theories models the sandwich panel as a beam. It is assumed that the beam only curves in one direction (cylindrical bending), which is the case for beams which are small compared to the span.

- Sandwich plates. If the sandwich panel has a larger width (span ratio) or is supported on four sides, the deformations have to be described for two directions.
- Sandwich shells. In case of sandwich shells the shape of the panel is curved in one or two directions. The number of available theories dealing with this type of sandwich panel is limited.

The following phenomena which are of importance for the mechanical behaviour of the overall geometry of the sandwich panel, should be taken into account during design:

- bending;
- global buckling;
- vibration.

In some references also attention is paid towards postbuckling behaviour and large deflections.

2.2 Local geometry

An important issue in designing a sandwich panel is the detailing of local geometry. Due to the localised loads and local changes in stiffness and strength high stresses and strains might occur. Examples of local aspects are, see also figure 2:

- point or line loads;
- support regions;
- edge connections;
- inserts, screws, rivets and top hats;
- holes and openings;
- stiffeners;
- diaphragms;
- delamination regions;
- tapered sandwich (varying thickness).

From practical applications and theoretical studies it is known that the detailing of local geometry is of great importance. Locally high stresses and strains occur, which are in many cases decisive. For this reason special attention should be paid towards possible local failure modes.

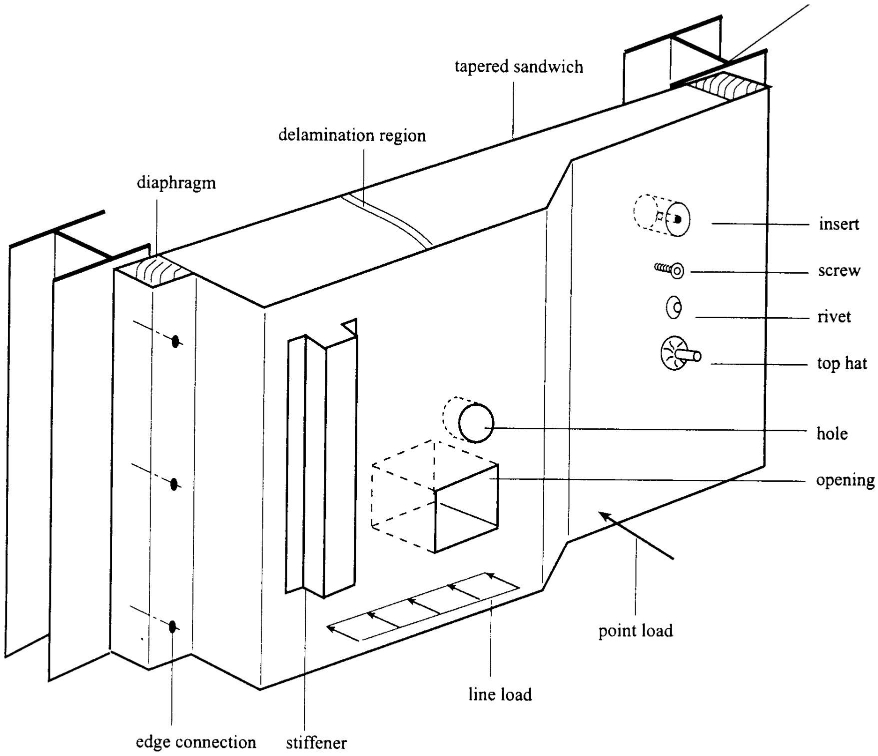


Figure 2 - Local geometries.

2.3 Failure modes

Considering the mechanical behaviour of sandwich panels, the following failure modes under static loading should be taken into account, see figure 3:

- failure of the face (yielding or fracture);
- wrinkling and dimpling of the face;
- shear failure of the core material;
- shear crimping of the core material (instability phenomenon);
- overall buckling (and interaction effects with local failure modes);
- delamination of the interface between the core and face;
- long-term creep;
- overall and local deflections.

Besides these failure modes also the possibility of fatigue failure due to cyclic loads and dynamic effects due to vibration or impact loads, should be taken into account.

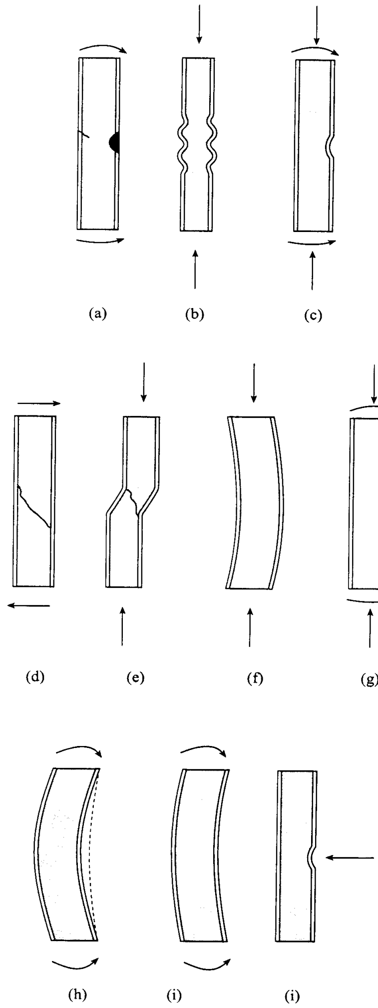


Figure 3 - Failure modes. (a) failure of the face: yielding or fracture, (b) wrinkling of the face, (c) dimpling of the face, (d) shear failure of the core material, (e) shear crimping of the core material, (f) overall buckling, (g) delamination of the interface between the core and face, (h) long-term creep, (i) overall deflection and (j) local deflection.

3. CLASSIFICATION OF SANDWICH THEORIES

3.1 Overview

Up till now no well established definition of categories of theories for sandwich panels is available. Following the discussions of experts like Frostig and Thomsen, three main categories can be distinguished:

- classical theories;
- superposition approaches;
- higher-order theories.

All these theories and approaches are based on a three-layer concept. They give a mathematical description of the deformation of the sandwich panel. Each theory makes assumptions in modelling the behaviour of the core, the faces and the interaction between both. This will result into a set of differential equations. Most of the theories discussed in this paper assume small deformations, which results into linear differential equations. If larger deformations occur the non-linear behaviour should be described by non-linear differential equations. In literature only for the simplest cases closed-form solutions are given. For the more complex cases numerical solving techniques have to be used.

The three distinguished categories will be discussed in the following sections. Additional attention will be paid towards theories used for finite element methods.

3.2 Classical theories

The so-called classical theories have been mainly developed during the period after the Second World War. The work of a number of researchers (e.g. of the Forest Product Laboratories of United States Forest Service, Reissner, Libove and Batdorf, Hoff, and Mindlin) have been collected by Plantema of the N.L.R., The Netherlands [2], by Allen of the University of Southampton, United Kingdom [3] and by Stamm and Witte of Hoesch, Germany [4]. A present-day interpretation is given by Zenkert [6], and Allen has made additional remarks over the years, see e.g. [7].

The classical theories make use of the following basic assumptions:

- No transverse flexibility of the core material occurs, which means that the deflection of the upper and lower faces are equal to each other. This is also known as the "antiplane" concept.
- The longitudinal displacement distribution through the height of the core is linear.

The total displacement of the sandwich panel is split into two parts, as indicated in figure 4. In the primary deformation the sandwich panel behaves like a normal beam without shear deformation, while in the secondary deformation the faces bend about there own neutral axis and the core deforms under shear.

In [2], [3], [4] and [6] differential equations are derived for sandwich beams, plates and shells in case of bending, overall buckling, wrinkling and vibration. For some simple cases of loads and supports, closed-form solutions are formulated.

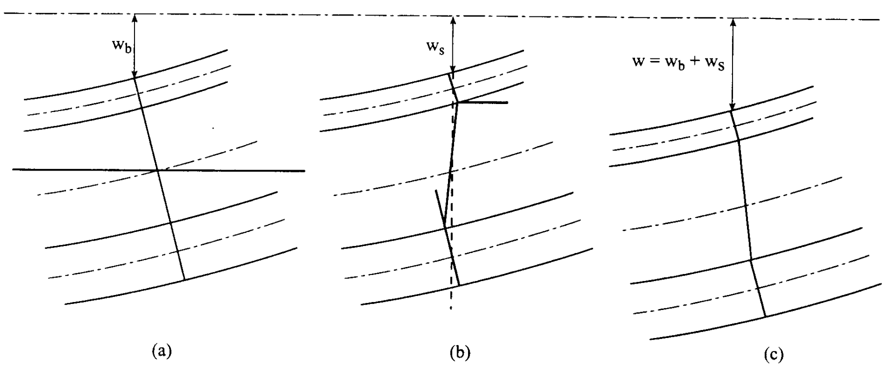


Figure 4 - Displacement according to classical theories. (a) Primairy deformation as a beam, (b) secundaire deformation due to shear, and (c) total displacement.

3.3 Superposition approaches

To study the effects of the local geometry some researchers have proposed so-called superposition approaches. In these approaches the local effects are formulated seperately and superposed upon a solution of the classical theories describing the overall behaviour of the sandwich panel.

One of the first superposition approaches considered in literature is the description of the wrinkling of the face under a compressive load. In [2], [3], [4] and [6] attention is paid towards this local failure mode. The face under compression according to a classical theory, is modelled as an elastically supported beam, as is shown in figure 5. With this approach the wavelength of the buckle is determined and a practical formula for the buckling stress is derived.

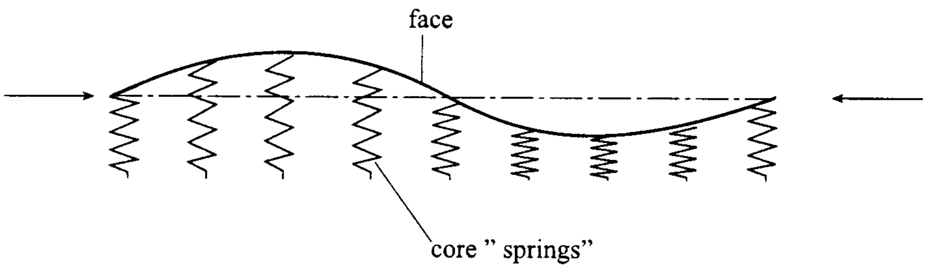


Figure 5 - Wrinkling of the face modelled by an elastically supported beam.

In [8] to [14] superposition approaches are worked out for localised loads. Sandwich beams with equal and unequal face thicknesses are considered in [8] and [9]. The solution is based on superposition of two types of beam behaviour, as is shown in figure 6. The first type describes the overall beam behaviour on basis of the anti-plane approach. The second type describes local behaviour due to e.g. local loads by modelling the core material as continuously distributed linear tension-compression springs. In [10] to [14] on the other hand a so-called two-parameter foundation model is used. This model also includes shear interaction effects. Beside sandwich beams discussed in [10] to [13], in [14] also a circular sandwich plate is considered.

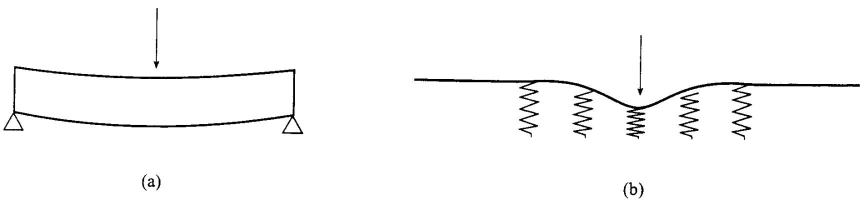


Figure 6 - Superposition approach of a sandwich beam with a localized load. (a) Overall beam behaviour on basis of the anti-plane approach and (b) local behaviour on basis of an elastic foundation model.

Most of the derivations of these superposition approaches results into a general solution with a number of unknown integration constants. The determination of the values of these constants under given boundary conditions is in most cases too complicated by analytical methods. An appropriate numerical algorithm to solve these unknown integration constants might be an alternative, but it is probably easier to solve the derived differential equations together with the statement boundary conditions numerically.

3.4 Higher-order theories

In case of a sandwich panel with a transversely flexible core the assumptions of an anti-plane sandwich and linear section planes of the core after deformations are no longer valid for local circumstances. This does not only mean that the classical theories can not be used for this case, but also that superposition approaches ignore certain effects. An other comment on classical theories is that no proper description of the boundary conditions is made. Classical theories assume that the boundary conditions are the same for the entire height of the section, which is not very realistic for practical applications. To take both the non-linear displacement fields of the core material and realistic supports into account, higher-order theories have been developed during the end of the eighties and nineties [15]. In principle these higher-order theories can be seen as extensions of the classical theories and superposition approaches discussed in the preceding sections.

Higher-order theories have the ability to model both sandwich beams as well as sandwich panels. The upper and lower face might be made of metallic or composite (un-)symmetrically laminated material, with (non-)identical mechanical and geometric properties. The core material might be made of foam, honeycomb, wood or mineral wool. The formulations of theories use well-known beam or plate theories for the faces and the elasticity theory for the core. By using these formulations, the higher-order effects caused by the non-linearity of the longitudinal and the transverse deformations of the core through the height are included. These effects are schematically illustrated in figure 7. The results are presented in terms of internal resultants and displacements of the faces, peeling and shear stresses into the interface between faces and core, and stresses and displacements of the core. These results are also available for local geometry. The theories can be formulated such that any type of loading exerted to the faces and any type of boundary or continuity conditions are handled.

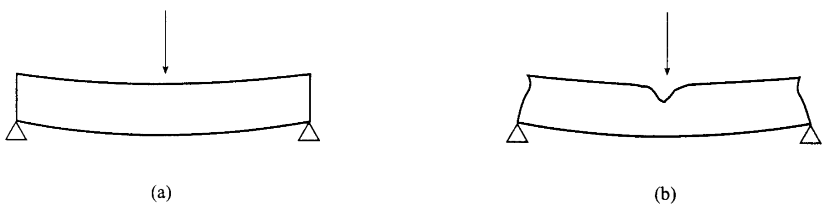


Figure 7 - High-order theory of a sandwich beam with a localized load. (a) Overall beam behaviour on basis of the anti-plane approach and (b) higher-order deformations caused by non-linearity of the longitudinal and the transverse deformations of the core through the height.

The derivations presented and discussed in [16] to [22] for sandwich beams, consider the faces as ordinary beams, which are interconnected through equilibrium and compatibility at the interface layers with the core. The core is considered to be a two-dimensional elastic medium. Different boundary conditions and continuity requirements for the two faces are allowed and different loading may be applied on the faces. With the theories presented in [16] to [22] it is possible to analyse sandwich beams with:

- point loads and point support regions [16];
- edge and inner delamination regions [17];

- overall buckling behaviour [18];
- edge and inner transverse diaphragms [19];
- unsymmetrical laminated composite faces [20];
- cut-off edge connection [21].

To illustrate these local geometry, an overview is given in figure 8. In [21] a summary of these theories is given, while in [22] the results are verified experimentally for the case of a three-point loaded sandwich beam.

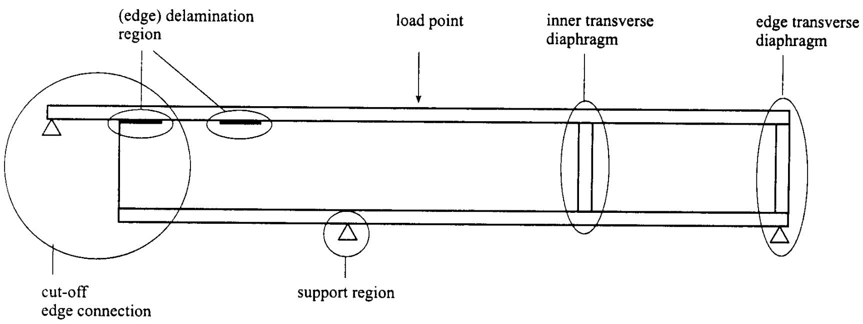


Figure 8 - Overview of sandwich beam with local geometries for which higher-order theories are developed in [16] to [22].

A further development of higher-order theories is made in [23] and [24] for sandwich panels in two directions:

- point loads and point support regions [23];
- overall buckling behaviour [24].

Also for the application of inserts in sandwich plates, see figure 9, the higher-order theories are worked out. The derivations are presented and discussed in [25] to [28]. In [25] the theory for sandwich plates with through-the-thickness inserts is presented, while in [26] the analyses for sandwich plates with fully potted and partly potted inserts are presented. The reports [27] and [28] give further background information about these derivations.

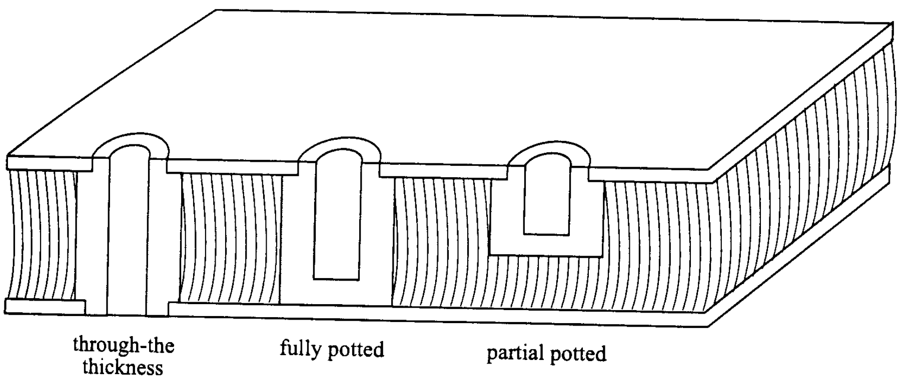


Figure 9 - Overview of sandwich plate with inserts for which higher-order theories are developed in [25] to [28].

The results of the derivations presented in [16] to [28] are a set of differential equations with the statement of boundary conditions. This is also called a boundary value problem. For only a limited number of applications this problem can be solved analytically; mostly a numerical procedure has to be used. First the differential equations should be reformulated into a set of first-order differential equations. Than the boundary value problem should be transformed into a set of interconnected initial value problems. The solution can be find by direct integration. Thomsen suggests in [25] to [28], to use the so-called "multi-segment method of integration" [29]. For implementation of this numerical procedure see [30].

3.5 Finite-element methods

Most of the finite-element methods for sandwich panels are also based on two-dimensional plate and shell models. In [5] and [31] the following classification of two-dimensional finite-element approaches is made:

- Global approximation models. The sandwich is replaced by an equivalent single-layer plate or shell element, with global through-the-thickness approximations for the displacements, strains and/or stresses. The models describe the core behaviour either with a first-order shear deformation approach or a higher-order approach. To apply models based on the first-order approach, shear correction factors have to be determined a priori.
- Discrete three-layer models. The sandwich is divided into three (or more) layers. For each layer approximations are made for the response quantities in the thickness direction. The models are based on either the classical theories discussed in section 3.2 or the higher-order theories discussed in section 3.4.
- Predictor-corrector approaches. These approaches make use of iterational processes. The information obtained in the first (predicting) phase of the analysis is used to correct the model to improve the response.

Additional to this list of two-dimensional finite-element approaches, also the following approaches are discussed in [5]:

- Detailed three-dimensional models of sandwich panels for which e.g. the honeycomb core material and laminated faces are fully modelled.
- Three-dimensional and quasi-three-dimensional models for which the core is modelled by an equivalent solid elements and the faces are modelled by equivalent continuum, plate or shell elements.
- Simplified models to study specific behavioural modes of sandwiches, like global buckling, panel buckling, face wrinkling or dimpling.

Of these approaches mostly the three-layer model is included in available finite element packages. For more detailed studies of local geometry, three-dimensional modeling of the sandwich panel with standard elements (solids, plates and shells) is used.

A detailed discussion of the available finite-element approaches is out of the scope of this paper. In [5] an extended overview of references is given following the classification mentioned above. This overview deals with the following topics:

- Stress analyses of sandwiches with various geometry and core configurations subjected to mechanical, thermal and hygrometric loading (table 2 of [5]).
- Free-vibration analyses and in some cases also damping analyses (table 3 of [5]).

- Transient dynamic and impact response analyses, considering forced vibration response, wave propagation and dynamic buckling (table 4 of [5]).
- Global (table 5a of [5]) and local buckling analyses, like face wrinkling and core crimpling (table 5b of [5]).
- Large deflection and post-buckling problem analyses (table 6 of [5]).
- Effect analyses of discontinuities like holes, cutouts, stiffeners, damages, and geometrical changes like tapered thickness (table 7 of [5]).

This overview is extended with a large number of references dealing with concepts, process developments, applications, design and optimisation, plasticity, and impact damage and tolerances (table 8 of [5]).

4. THEORIES USED FOR THE BUILDING SECTOR

Of the three main "classical" text books about sandwich panel theories mentioned in chapter 1, those written by Plantema [2] and Allen [3] are mainly based on experience of the aerospace industry. The book written by Stamm and Witte [4] on the other hand also focuses on building applications. Together with the knowledge gathered within three Ph.D. research projects [32], [33] and [34], carried out at the University of Darmstadt, Germany, during the seventies, these references can be regarded as the basis of the design rules for building applications used nowadays.

Additional to the theories described in [2] and [3], Stamm and Witte [4] discussed specific topics important for building applications, see figure 10:

- Load case of a temperature gradient.
- Profiled faces and profiled sandwiches.
- Fire resistance.
- Building physics.

In the thesis of Basu [32] the behaviour of the materials of a sandwich panel is studied. Special attention is given towards the properties of rigid foam core material. The thesis of Linke [33] deals with design calculations under short- and long-term loading. As known the creep behaviour of the core materials have to be taken into account. In the thesis of Berner [34] the influence of a temperature loading and the behaviour of sandwich panels under fire load conditions is studied.

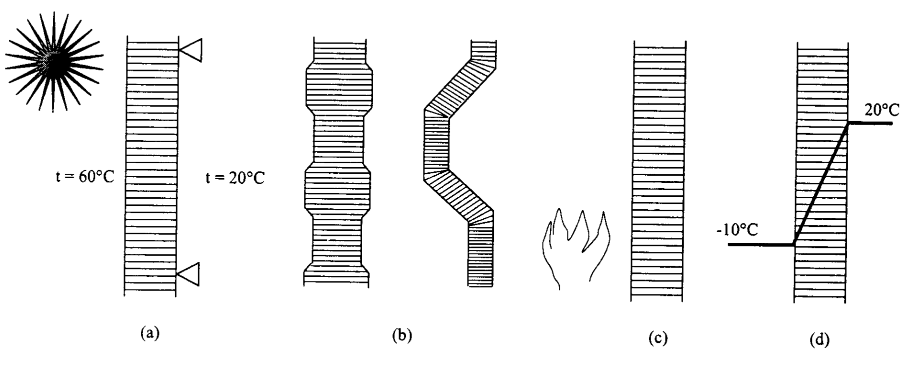


Figure 10 - Topics important for building applications. (a) Load case of a temperature gradient, (b) profiled skins and profiles sandwiches, (c) fire resistance and (d) building physics.

A comprehensive review of this German work is presented in chapter 7 and 8 of [35]. Chapter 7 pays attention towards sandwich panels made from profiled steel faces and a polyurethane foam core, while in chapter 8 mineral wool core is considered. These chapters also considers the safety philosophy known as the partial safety factor approach used nowadays for building applications.

Additional to the work carried out in Germany, also other researchers have studied sandwich panels with profiled faces during the last decades. An overview of the main results of these studies is performed by Davies [36]. Both closed-form solutions as well as finite element techniques are discussed. Beside the mentioned references an almost unlimited number of papers has been published over the years. Since this paper mainly focuses on the fundamental theories used for sandwiches, no further information is provided.

As a result of these studies a number of European recommendations have been drafted [37], [38], [39]. A recent state-of-the-art of the design rules used nowadays (in Germany) is presented in [40]. Additional design rules for sandwich panels with openings have been formulated in [41]. In view of the theories discussed in chapter 3 of this paper, it can be observed that the theories used for sandwich panels for building applications are mainly based on classical theories.

5. APPLICATION OF AVAILABLE SANDWICH THEORIES

5.1 Sandwich beam under 3-point bending load

To illustrate the possibilities and limitation of available sandwich theories, in this chapter an example of a sandwich beam under 3-point bending load is worked out. Both the overall behaviour as well as local behaviour near the load point and the support is considered. The set up of the example is shown in figure 11. The used dimensions are:

- beam length l_s = 600 mm;
- beam width b_s = 200 mm;
- sandwich height h_s = 80 mm;
- thickness upper face d_t = 1.0 mm;
- thickness lower face d_b = 0.7 mm.

The properties of the steel faces and heavy PS foam core are:

- Young's modulus faces E_f = 210000 N/mm^2;
- shear modulus core G_c = 20 N/mm^2;
- Young's modulus core E_c = 60 N/mm^2.

The applied load is equal to F = 3000 N. It is noted here that in this example the beam length is chosen such that local effects can be seen easily within the graphs. The complete calculations of this example are reported in [42].

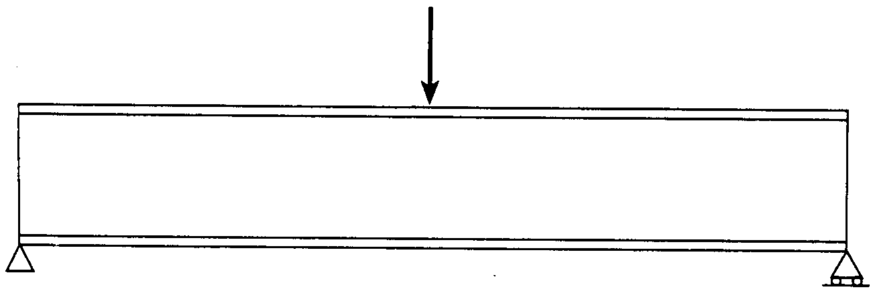


Figure 11 - Example of sandwich beam under 3-point bending.

5.2 Applied sandwich theories

In this example a classical theory, a superposition approach and a higher-order theory are compared. To simplify the problem of a sandwich beam under a 3-point bending load, only one half is considered, see figure 12.

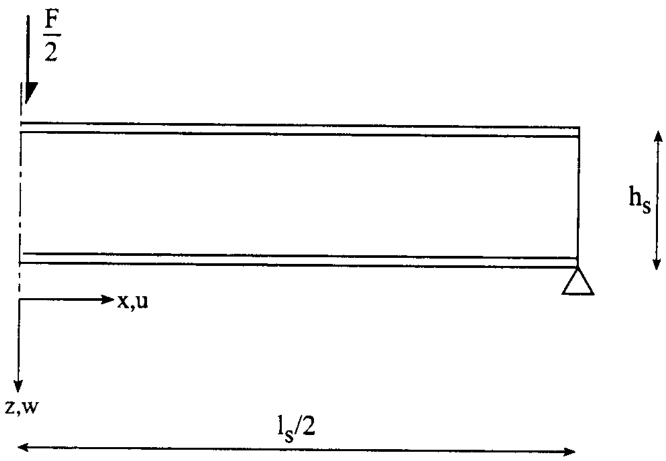


Figure 12 - Modelling of one half of the sandwich beam under 3-point bending.

The classical theory [3] applied in this example, assumes that the faces are thin compared to the sandwich height and that the core is weak compared to the faces. The deformations are assumed to be a superposition of two quite independent parts. The primary deformation takes into account the deformation according to the ordinary beam theory. The secondairy deformation takes into account the contribution of the shear deformation of the core. The derived differential equations are:

$$D_s \frac{d^4 w_b}{dx^4} = 0 \qquad \text{(ordinary beam theory)} \qquad (1a)$$

$$\frac{dw_s}{dx} = \frac{-F/2}{A_s G_c} \qquad \text{(shear deformation)} \qquad (1b)$$

where:

$$D_s = \frac{b_s E_f d_t E_f d_b d^2}{E_f d_t + E_f d_b} \quad (2a)$$

$$A_s = b_s d \quad (2b)$$

$$d = \frac{d_t}{2} + c + \frac{d_b}{2} \quad (2c)$$

$$c = h_s - (d_t + d_b) \quad (2d)$$

In these equations w_b and w_s are the vertical displacements due to bending respectivily shear deformations. D_s is the flexural rigidity and A_sG_c is often refered are the shear stiffness of the sandwich beam. The boundary conditions for $x = 0$ and $x = l_s/2$ are:

$\varphi_b(x = 0) = 0$ (3a)

$w_b(x = l_s/2) = 0$ (3b)

$D(x = l_s/2) = -F/2$ (3c)

$M(x = l_s/2) = 0$ (3d)

$w_s(x = l_s/2) = 0$ (3e)

In these boundary conditions φ_b is the rotation of the beam, D is the shear force and M is the bending moment. This boundary value problem can be solved analytical. The results for the deflection due to bending and shear are respectivily:

$$w_b(x) = \frac{F}{12D_s}x^3 - \frac{Fl_s}{8D_s}x^2 + \frac{Fl_s{}^3}{48D_s} \quad (4a)$$

$$w_s(x) = \frac{-F}{2A_sG_c}x + \frac{Fl_s}{4A_sG_c} \quad (4b)$$

The shear forces, bending moments, shear stresses and membrane stresses can be found with use of the derived constitutive relations.

The superposition approach applied in this example models local effects, which have to be superposed on the solution of the classical theory discussed above. The local effects are modelled with the simplest possible model, known as the Winkler foundation model, see also chapter 12 of [6]. In this model the supporting medium is modelled as continuously distributed linear tension/compression springs. The elastic response is expressed as follows:

$$q_z(x) = -K_z w(x) \quad (5a)$$

where $q_z(x)$ is the interfacial transverse normal stress per unit width and K_z is the foundation modulus. Its value is given by the following emperical expression:

$$K_z = 0.28E_c\sqrt[3]{\frac{E_c}{D_f}} \quad (5b)$$

where D_f is the flexural rigidity of the considered face per unit width. The differential equation for this problem is:

$$\frac{d^4w}{dx^4} + 4\kappa^4 w = 0 \quad (6a)$$

The additional parameter κ, known as the inverse of the characteristic length, is equal to:

$$\kappa^4 = \frac{K_z}{4D_f} \tag{6b}$$

In the considered example two regions with local effects can be distinguish. For the load point region the boundary conditions for $x = 0$ and $x = l_s/2$ are:

$$\varphi(x = 0) = 0 \tag{7a}$$

$$D(x = 0) = F/2 \tag{7b}$$

$$M(x = l_s/2) = 0 \tag{7c}$$

$$D(x = l_s/2) = 0 \tag{7d}$$

For the support region on the other hand the boundary conditions are:

$$M(x = l_s/2) = 0 \tag{8a}$$

$$D(x = l_s/2) = F/2 \tag{8b}$$

$$M(x = 0) = 0 \tag{8c}$$

$$D(x = 0) = 0 \tag{8d}$$

This boundary value problem can be solved analytical. Since the solutions of both cases are rather complicated, the equations are not presented here.

As mentioned in section 3.4, the higher-order theory [16] applied in this example considers the faces as ordinary beams, which are interconnected through equilibrium and compatibility at the interface layers with the core. The core is considered to be a two-dimensional elastic medium. The differential equations for this example are:

$$E_f A_t \frac{d^2 u_{ot}}{dx^2} + b_s \tau = 0 \tag{9a}$$

$$E_f A_b \frac{d^2 u_{ob}}{dx^2} - b_s \tau = 0 \tag{9b}$$

$$E_f I_t \frac{d^4 w_t}{dx^4} + \frac{b_s E_c}{c} w_t - \frac{b_s E_c}{c} w_b - \frac{b_s (c + d_t)}{2} \frac{d\tau}{dx} = 0 \tag{9c}$$

$$E_f I_b \frac{d^4 w_b}{dx^4} + \frac{b_s E_c}{c} w_b - \frac{b_s E_c}{c} w_t - \frac{b_s (c + d_b)}{2} \frac{d\tau}{dx} = 0 \tag{9d}$$

$$b_s u_{ot} - b_s u_{ob} - \frac{b_s (c + d_t)}{2} \frac{dw_t}{dx} - \frac{b_s (c + d_b)}{2} \frac{dw_b}{dx} - \frac{b_s c^3}{12 E_c} \frac{d^2 \tau}{dx^2} + \frac{b_s c}{G_c} \tau = 0 \tag{9e}$$

In these equations u_{ot} and u_{ob} are the horizontal displacements of the upper respectivily lower face, τ is the shear stress in the core material and w_t and w_b are the vertical displacements of the upper respectivily lower face. The additional parameters are equal to:

$$A_t = d_t b_s \tag{10a}$$

$$A_b = d_b b_s \tag{10b}$$

$$I_t = \frac{1}{12} b_s d_t^{\ 3} \tag{10c}$$

$$I_b = \frac{1}{12} b_s d_b^{\ 3} \tag{10d}$$

$$c = h_s - (d_t + d_b) \tag{10e}$$

The boundary conditions are given for the upper face, the lower face and the core near the point load region ($x = 0$) and the support region ($x = l_s$), as follows:

$$u_{ot}(x = 0) = 0 \tag{11a}$$

$$\varphi_t(x = 0) = 0 \tag{11b}$$

$$\frac{dM_t(x=0)}{dx} + \frac{b_s d_t}{2} \tau(x=0) = \frac{-F}{2} \tag{11c}$$

$$u_{ob}(x = 0) = 0 \tag{11d}$$

$$\varphi_b(x = 0) = 0 \tag{11e}$$

$$\frac{dM_b(x=0)}{dx} + \frac{b_s d_b}{2} \tau(x=0) = 0 \tag{11f}$$

$$\tau(x = 0) = 0 \tag{11g}$$

$$N_{ot}(x = l_s/2) = 0 \tag{11h}$$

$$M_t(x = l_s/2) = 0 \tag{11i}$$

$$\frac{dM_t(x = l_s/2)}{dx} + \frac{b_s d_t}{2} \tau(x = l_s/2) = 0 \tag{11j}$$

$$\varphi_b(x = l_s/2) = 0 \tag{11k}$$

$$M_b(x = l_s/2) = 0 \tag{11l}$$

$$w_b(x = l_s/2) = 0 \tag{11m}$$

$$\tau(x = l_s/2) = 0 \tag{11n}$$

In these boundary conditions φ_t and φ_b are the rotation of the upper respectivily lower face, M_t and M_b are the bending moments.

In [16] an analytical solution is discussed, but a numerical procedure is necessary to solve the integration constants. Instead of doing this, in this example the above given boundary value problem is rewritten into a set of linear first order differential equations, which is solved with the numerical procedure described in [29] and [30]. The results are discussed in the following sections.

5.3 Comparison of deflections

The deflections calculated according to the classical theory, equation (4), are presented in figure 13. On the horizontal axis the x-coordinates with x = 0 mm at the point load and x = 300 mm at the support, the right half of the sandwich beam is given. See also figure 12. On the vertical axis the deflections due to bending and shear deformations are given. Note that the positive values of the deflections represents a downwards movement of the sandwich beam. It is observed that for this example the deflections are dominated by shear.

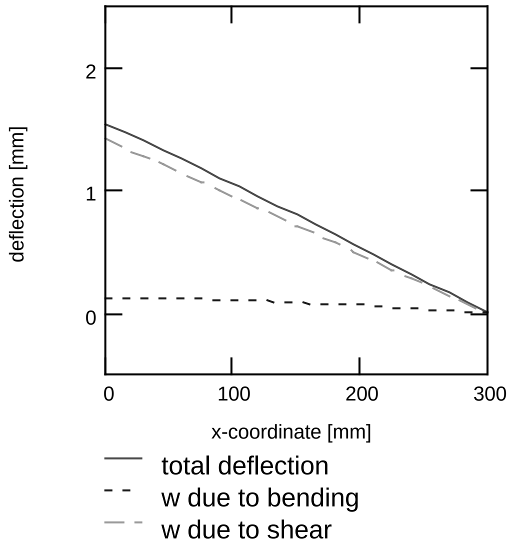


Figure 13 - Deflections according to classical theory.

To study local effects near the point load and the support, the superposition approach or the higher-order theory has to be used. In figure 14a the local effects calculated with the Winkler foundation model superposed upon the deflections according to the classical theory, are presented. In figure 14b the deflections of both the upper and lower face of the sandwich beam calculated according to the higher-order theory, are presented. Both results show the significant influence of the local forces on the deflections. Only the superposition approach is not completely realistic, because the deflections of the lower face near the support region (x = 300 mm) are negative.

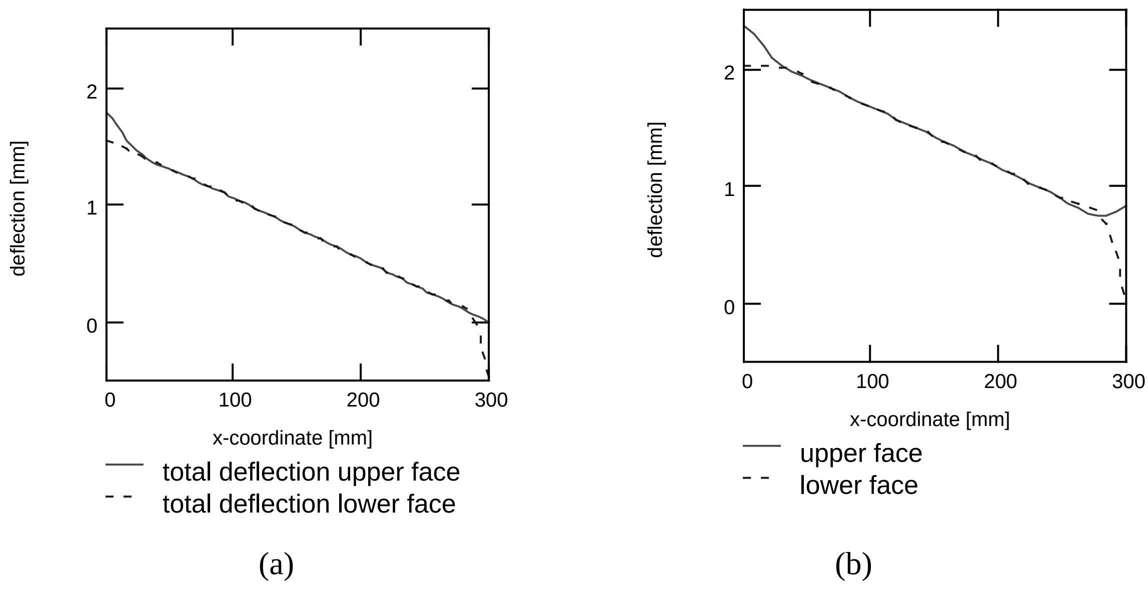


Figure 14 - Deflections according to: (a) superposition approach, (b) higher-order theory.

To illustrate the advantage of the higher-order theory, the calculated vertical and horizontal deformations of the core material are plotted in figure 15. The significant difference between the load point region and the support region is caused by the fact that the upper face near the point load region is continuous, while the lower face near the support ends. This causes stronger deformations of the core and faces in the support region.

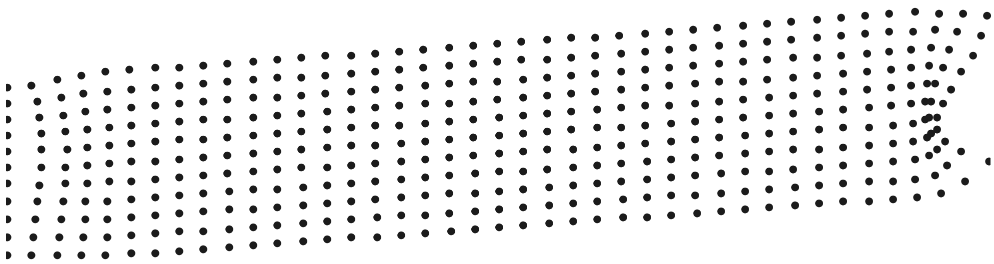
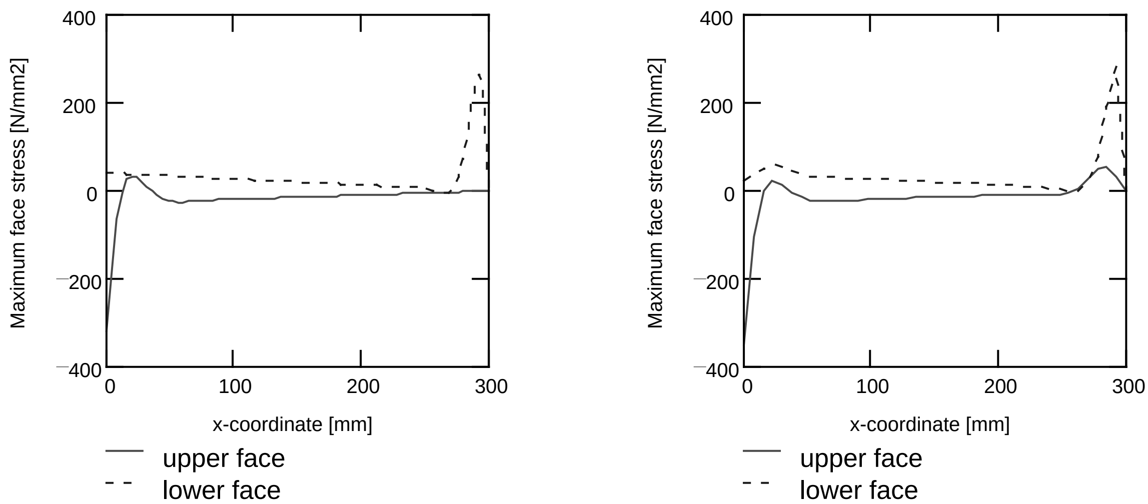


Figure 15 - Deformations (magnified) of the core calculated with the higher-order theory. The load and the support regions are on the left respectivily right side of the figure.

5.4 Comparison of stress distributions

The maximum stresses in upper and lower faces due to overall bending of sandwich beam and local bending of faces, are presented in figure 16. It is observed that near the load point and support regions the local bending stresses are high compared with the membrane stresses due to overall bending. The magnitude of stresses according to the superposition approach and according to the higher-order theory are the same. The advantage of the higher-order theory is that it is possible to investigate the effects in the unloaded face due to a force active on the opposite face.



(a) (b)

Figure 16 - Maximum stresses in the faces. (a) Results according to superposition approach, (b) results according to higher-order theory.

The distribution of the shear stresses calculated with the superposition approach and the higher-order theory, are presented in figure 17. With the superposition approach it is not possible to calculate local effects on the shear stress distribution, which means that the calculated values are fully based on the classical theory. Only the higher-order approach could show the influence of local effect on the shear stress distribution.

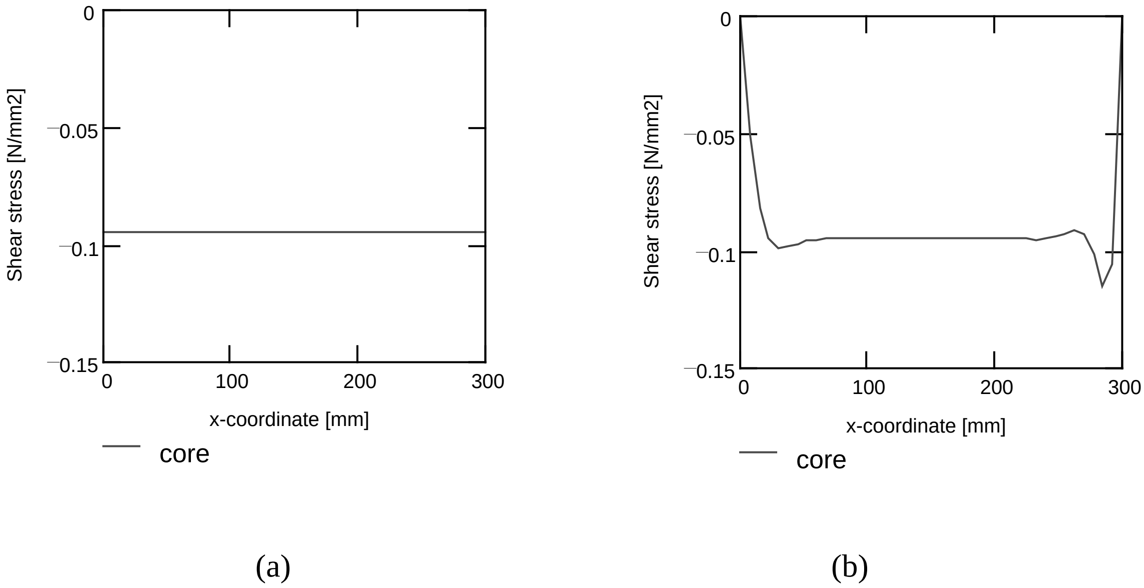
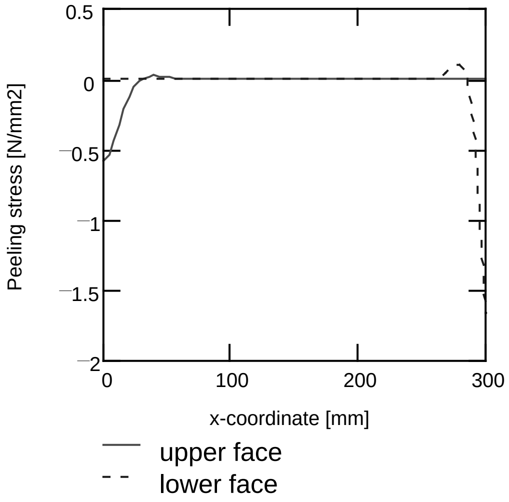
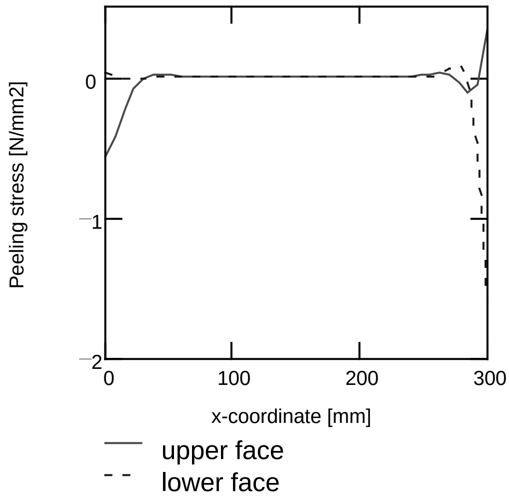


Figure 17 - Shear stresses in the core. (a) Results according to superposition approach, (b) results according to higher-order theory.

The calculated peeling stresses in the interface between the core and the faces are presented in figure 18. Note that the negative values are compressive stresses. The calculated compressive stress of about 1.5 N/mm^2 near the support, is rather high for the used PS foam core. An other important observation is that only the results of the higher-order theory show that the adhesive bond layer between core and upper face in the support region is loaded by tensile stresses. This might cause failure within the bond line.



(a)



(b)

Figure 18 - Peeling stresses in the interface. (a) Results according to superposition approach, (b) results according to higher-order theory.

5.5 Discussion

The example discussed in the preceding sections shows that the overall behaviour of a sandwich beam can be predicted rather well with the classical theory. To study local effects on the other hand, superposition approaches or higher-order theories have to be used. The examples shows that the higher-order theory gives the best descriptions, since it takes influences through the thickness of the sandwich into account. From this point of view the higher-order theory is preferable. The only disadvantage is that the derivations are more complex than those for the superposition approach. But since general numerical procedures are developed to solve the boundary value problem easily, the higher-order theory is preferable.

6. FINAL REMARKS

In this paper a comprehensive overview of theories for sandwich panels is presented. Special attention is given towards analytical solutions which might be useful for practical applications. Classical theories can be used to calculate the overall behaviour of sandwich panels, while superposition approaches or higher-order theories have to be used to calculate local effects. The presented example indicates that higher-order theories are preferable.

Additional information is given about the use of finite-element methods. The advantage of these methods is that an almost unlimited number of configarations can be modeled. The higher-order theories on the other hand considers only a limited number of configurations, but their advantage is that it is much easier to generate an answer.

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